STYRELSEN FÖR

VINTERSJÖFARTSFORSKNING

WINTER NAVIGATION RESEARCH BOARD

Research Report No 93

Arto Sorsimo, Tapio Nyman and Jaakko Heinonen

SHIP-ICE INTERACTION IN A CHANNEL

Finnish Transport Safety Agency	Swedish Maritime Administration
Finnish Transport Agency	Swedish Transport Agency
Finland	Sweden

Talvimerenkulun tutkimusraportit — Winter Navigation Research Reports
ISSN 2342-4303
ISBN 978-952-311-164-6

FOREWORD

In this report no 93, the Winter Navigation Research Board presents the results of the research project SINTERA. Discrete element method (DEM) was used to simulate the ice resistance of a ship. The channel profile was modelled according to the Finnish-Swedish ice class rules and the ice blocks were modelled as spherical particles.

The simulations produced visually realistic results of ship-ice interaction in an ice channel but the resistance predictions were significantly lower than those given by the analytical model in the ice class rules. Main problems in the performed DEM simulations were identified to be material properties of the ridge rubble and higher porosity and thus lower cohesion of the simulated rubble when compared with observations.

DEM as such was found to be a promising methodology for further work.

The Winter Navigation Research Board warmly thanks Mr. Arto Sorsimo, Mr. Tapio Nyman and Mr. Jaakko Heinonen for this report.

Helsinki and Norrköping

October 2016

Jorma Kämäräinen

Finnish Transport Safety Agency

Markus Karjalainen

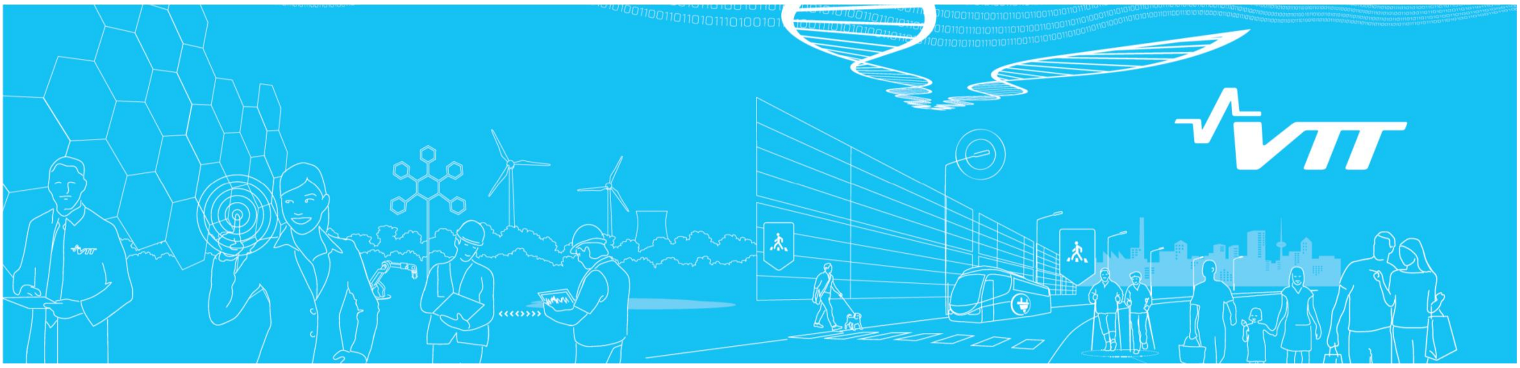
Finnish Transport Agency

Tomas Årnell

Swedish Maritime Administration

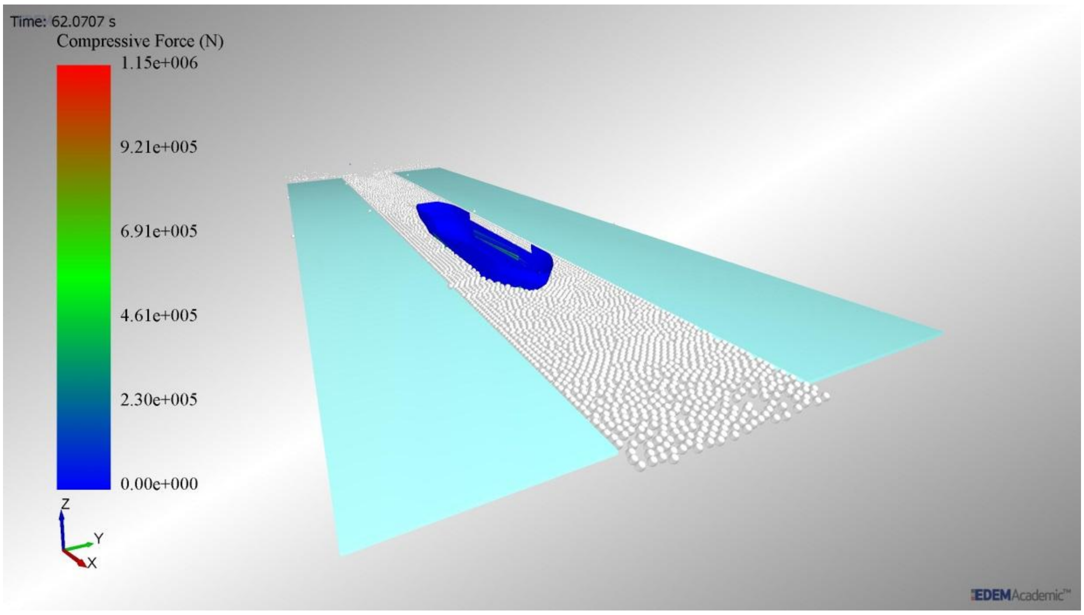
Stefan Eriksson

Swedish Transport Agency



RESEARCH REPORT

VTT-R-05953-14



Ship-ice interaction in a channel

Authors: Arto Sorsimo, Tapio Nyman, Jaakko Heinonen

Confidentiality: Confidential

Report's title

Ship-ice interaction in a channel

Customer, contact person, address	**Order reference**
Finnish Transport Safety Agency (Trafi) Regulation/Transport Technology Jorma Kämäräinen, Chief Adviser Kumpulantie 9, 00520 Helsinki P.O.Box 320, FI-00101 Helsinki, Finland	W14-16 SINTERA
Project name	**Project number/Short name**
Ship-ice interaction in a channel	86112 SINTERA
Author(s)	**Pages**
Arto Sorsimo, Tapio Nyman, Jaakko Heinonen	16
Keywords	**Report identification code**
ice resistance, brash ice, channel, ship, DEM	VTT-R-05953-14

Summary

The discrete element method was applied to simulate the ice resistance of ship in a channel with brash ice by using EDEM 2.6.0 software. Real ship geometry of Lunni was modelled. In the simulation, the ship was moving unidirectionally in a brash ice channel, which geometry was modelled according to Finnish-Swedish ice class rule. Brash ice in the channel was modelled by a huge number of ice particles with a spherical shape interacting with each other and with the ship.

The simulation of brash ice failure process was visually realistic corresponding to the full-scale observations. However, the channel resistance was substantially lower compared to the analytical study. The major lack in the model relates to the model of brash ice and the evaluation of the material parameters. Current parameterization refers to the ice rubble in ridges. As the brash ice in the channel was created by unisize spherical ice particles, the porosity was higher than observed by experiments. That made brash ice looser and therefore weaker, which also caused that the implementation of the cohesive freeze bond between the particles didn't produce any notable effect on the channel resistance.

As the user can model real ship geometry and define the channel geometry and material parameters of ice without considerable restrictions, the DEM-methodology itself is a promising basis for further development.

The main needs for future research are focused to measure brash ice properties in full-scale. To understand material behaviour more in details is crucial for reliable channel resistance simulations.

Confidentiality	Confidential

Espoo 22.12.2014

Written by	**Reviewed by**	**Accepted by**
Jaakko Heinonen Principal scientist	Kari Kolari Senior Scientist	Johannes Hyrynen Research Area Manager

VTT's contact address

P.O. Box 1000, FI-02044 VTT, Finland

Distribution (customer and VTT)

Client, 1 original
VTT, 1 copy + 1 original to archive

Preface

This research project was funded 2014 by the Finnish-Swedish Winter Navigation Research Board. The target was to improve the scientific background for the ice resistance of ship in a channel with brash ice.

The authors would like to acknowledge the Finnish Transport Safety Agency, the Finnish Transport Agency, the Swedish Maritime Administration and the Swedish Transport Agency for the funding and for constructive discussions during the project.

Espoo 22.12.2014

Arto Sorsimo, Tapio Nyman, Jaakko Heinonen

Contents

1. Introduction and objectives

1.1 Background

The ice conditions in the Baltic Sea can vary significantly as can be seen from the ice chart in Figure 1. In the central part of the Gulf of Bothnia and Gulf of Finland the ice cover is broken and ridged by winds. Near the coast-line the islands prevent the ice field from the moving so the area of fast ice forms in front of the coast. The traffic which goes to the harbours has to go through the fast ice zone so a brash ice channel is developing to the fairway area.

Based on the observations made onboard vessels bound to the ports in the Gulf of Bothnia in winters 1991 [1], 1993 [2] and 1994 [3], the brash ice channel was the most common ice condition met by the vessels. This was one of the reasons why in the Finnish-Swedish Ice Class Rules the minimum required engine power shall be determined based on the brash ice resistance in old channel of the vessels planned to have some of the Finnish-Swedish ice classes IA Super, IA, IB or IC.

The calculation method for the brash ice resistance in old channel has been in the ice class rules since year 2002. However, there were no extensive full scale test results of the vessels built according to the rules. Thus the maritime administrations of Finland and Sweden invited research institutions and companies to apply from the Winter Navigation Research Board for financing of projects related to the ability of ships to navigate in ice, focusing on making full-scale tests of ships in a brash ice channel, or evaluation of correlation between model test results, calculations, rule requirements and full scale measurements related to the ability of ships to navigate in brash ice channels.

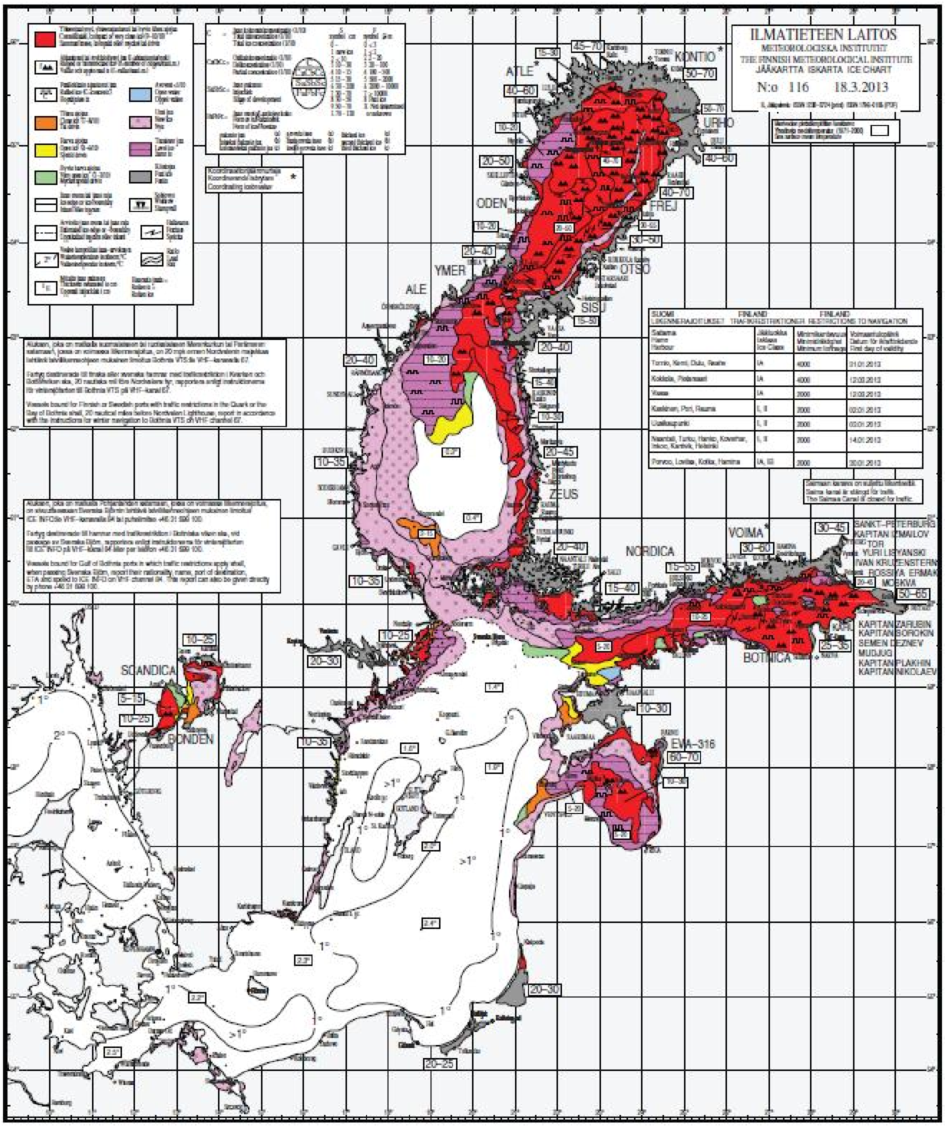


Figure 1. Ice condition in the Baltic Sea in March 2013 according to the ice chart published by the Finnish Meteorological Institute.

In the current project the discrete element method (DEM) has been applied in calculation of the brash ice resistance of a vessel in old channel having the same conditions as determined in the Finnish-Swedish ice class rules.

1.2 Goal

In this study the ice resistance of a ship in brash ice channel is examined. Two methods are compared against each other – analytical method and numerical method. The results from analytical method are based on the commonly used procedures for the calculation of ship brash ice resistance required by the Maritime Safety Regulation [4]. Numerical method that will be used is discrete element method, in which EDEM$^{©}$ software was used.

2. Numerical approach

2.1 Short description of DEM

Discrete element method is a numerical method used to compute the motion and forces for a large number of particles. The main difference between finite (FEM) and discrete element method is that in FEM a simulation typically consists continuum material within one geometry (particle) which is discretized with elements and the total effect, e.g. stress, is approximated from the nodal values of the mesh, while a DEM simulation typically consist of hundreds of particles and the main interest lies in the total effect of multiple particles into a specific geometry.

Due to the increased capability of computers, it is now possible to run simulations for millions of particles on a single processor. Nowadays DEM is widely used in solving engineering problems in granular and discontinuous materials. Some examples of applications are simulation-based design of bulk-materials handling and processing, process design for energy and power production and discrete manufacturing industry. Lately DEM has been applied also within ice mechanics; see for example [5], [6].

2.2 Methods

EDEM® 2.6.0 particle simulation software developed by DEM Solutions was used in this project.

2.2.1 Geometric models

Arctic product tanker Lunni was used as a geometric model of a ship, which was obtained from the customer, see Figure 2. Lunni represents a typical ship that is designed to operate in arctic conditions, as well as in the northern areas of the Baltic Sea. The velocity of the ship in the channel was chosen as 2.6 m/s (5 knots) corresponding the speed specified in the Finnish–Swedish ice class rules for calculations of the required engine power.

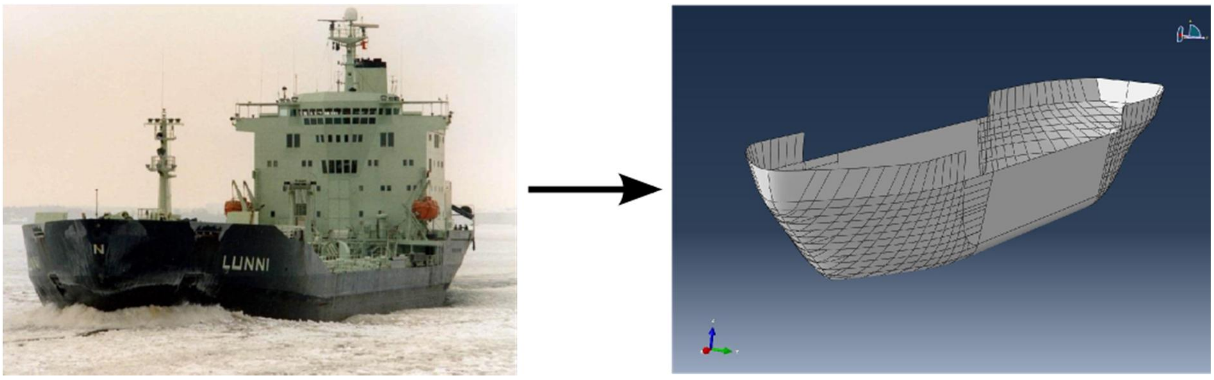


Figure 2. LUNNI Class tanker and its CAD-representation.

The general channel section is assumed to be similar as the illustration given in Figure 7. The geometry of the channel is determined according to the Finnish-Swedish ice class rules [7]. The width of the channel was fixed to 43 m and the thickness of the brash ice was assumed to be approximately 1 m. For full details of geometry parameters of the models, see Table 1.

Initial CAD-model of the ship was a halfway cut of the ship profile, which was mirrored in Abaqus CAE and then imported into EDEM. The movement of the ship was forced translation. The geometry of the channel was created in EDEM by placing fixed solid plates both sides of the channel representing surrounding ice field.

2.2.2 Particle models

Brash ice in the channel was modelled with discrete, sphere-shaped particles, for which the radius was fixed to 0.25m. This was assumed to represent the average shape and size of the ice blocks that is usually found in the brash ice channels during the spring. The contact forces between the particles were modelled with Hertzian contact that is the most convenient contact theory for sphere-shaped particles [8].

2.2.3 Modelling

The main objective of the simulations was to obtain value for the maximum compressive force subjected by the ice particles to the ship in the parallel direction with the translation of the ship. Before one can run a ship-channel interaction simulation, one must create the composition of the brash ice by placing particles randomly within the channel.

Multiple simulations were conducted which can be divided into two phases. The first phase was focused on the sanity-check of the boundary conditions and simulation parameters. In the second phase the simulations were run with verified parameters that were compared with analytical results.

Table 1. Parameters for the DEM simulation

Type	Parameter	Unit	Value	Source
General	Gravity	m/s^2	9.81	Approximated
	Water density	kg/m^3	1028	[9]
Geometry	Ship length	m	150	LUNNI .igs-model
	Ship width	m	21.5	LUNNI .igs-model
	Ship depth	m	9.5	LUNNI .igs-model
	Ship velocity	m/s	2.6	Approximated
	Channel length	m	500	Approximated
	Channel width	m	43	[7]
	Channel height	m	2	[7]
	Ice sheet height	m	1	[7]
	Ice particle radius	m	0.25	Approximated
Ship material	Density	kg/m^3	7750	Approximated
	Shear modulus	GPa	80	Approximated
	Poisson's ratio	-	0.30	Approximated
Ice material	Ice density	kg/m^3	920	[10]
	Poisson's ratio	-	0.35	[10]
	Elastic modulus	GPa	2.7	[10]
	Shear modulus	GPa	$G = E(2(1 + \boldsymbol{n}))^{-1}$	Derived
	Drag coefficient C_D	-	0.48	Approximated
Contact	Coefficient of Restitution (ice-ice)	-	0.8	[11]
	Coefficient of Static Friction (ice-ice)	-	0.5	[10]
	Coefficient of Rolling Friction (ice-ice)	-	0.001	Approximated
	Coefficient of Restitution (ice-ship)	-	0.6	Approximated
	Coefficient of Static Friction (ice-ship)	-	0.05	Approximated
	Coefficient of Rolling Friction (ice-ship)	-	0.001	Approximated
Freeze bond	Elastic stiffness (bond) $S_{B,n}$	N/m^3	1×10^9	[8]
	Shear stiffness (bond) $S_{B,t}$	N/m^3	$S_{B,t} = \boldsymbol{b} \times S_{B,n}$	[8]
	Shear stress factor $\boldsymbol{b}$	-	0.37	[8]
	Effective strength (part.-part.) $\boldsymbol{s}_{*,cr,p-p}$	Pa	$[1 \times 10^6, 1 \times 10^9]$	[8]
	Effective strength (part.-geom.) $\boldsymbol{s}_{*,cr,p-g}$	Pa	$\boldsymbol{s}_{*,cr,p-g} = 2 \times \boldsymbol{s}_{*,cr,p-p}$	[8]
	Bond disk scale	-	0.25	[8]
	Dissipation distance	mm	20	[12]
	Compression ratio	-	1	[8]

2.2.3.1 Validating simulation parameters

The first runs were focused on generating a reasonable geometry for the channel, see Figure 3. The channel geometry was difficult to establish merely by placing particles on the channel as the particles floated freely under the ice cover on the both sides of the channel. The solution was to insert barriers to both sides of the channel that represent ice blocks that are consolidated to the ice sheet, see Figure 3. Consequently the ice particles formed a pursued shape of the channel.

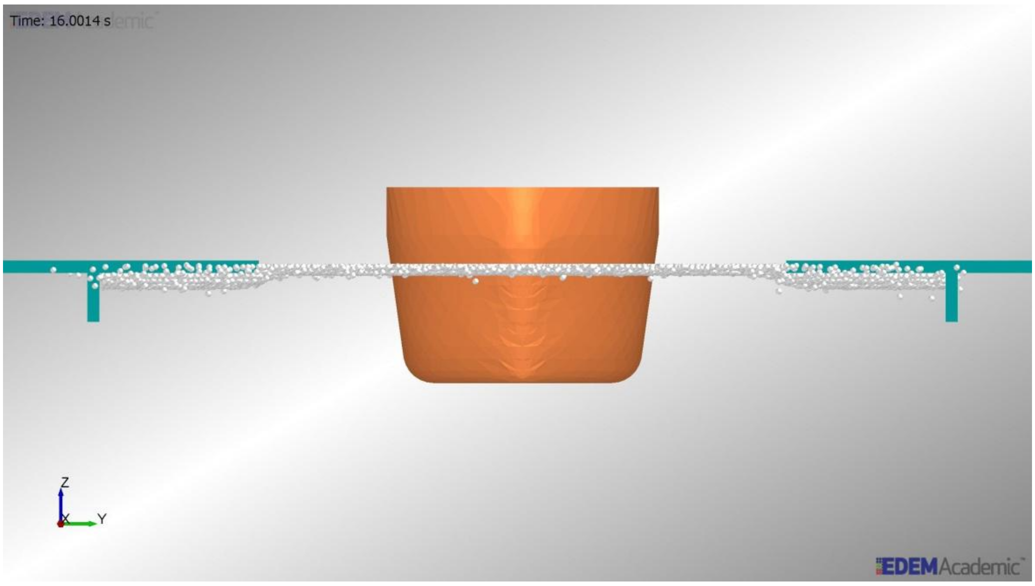


Figure 3. Channel profile before the translation of ship made with barriers on the both sides of the channel.

The length of the channel was investigated next. The goal was to determine sufficient length of the channel so that the inner and outer boundaries can be neglected with minimum length of the channel to minimize the simulation time. To understand this investigation fully, one should note that the inner and outer boundaries of the channel are open. When the particles are generated within the channel, the particles flow naturally outside of the channel from both ends due to high compaction of particles loaded with the buoyancy.

If the channel is long enough, one observes a steady state phase when the ship passes in the channel as shown in Figure 4. Note from Figure 4 that the velocity of particles on the both sides of the channel is approximately 0.25 m/s, which indicates that the particles are flowing out from the channel, while in the middle of channel the particles can be considered quasi-static. The undesirable flow of particles was later avoided by introducing freeze bonds between the particles.

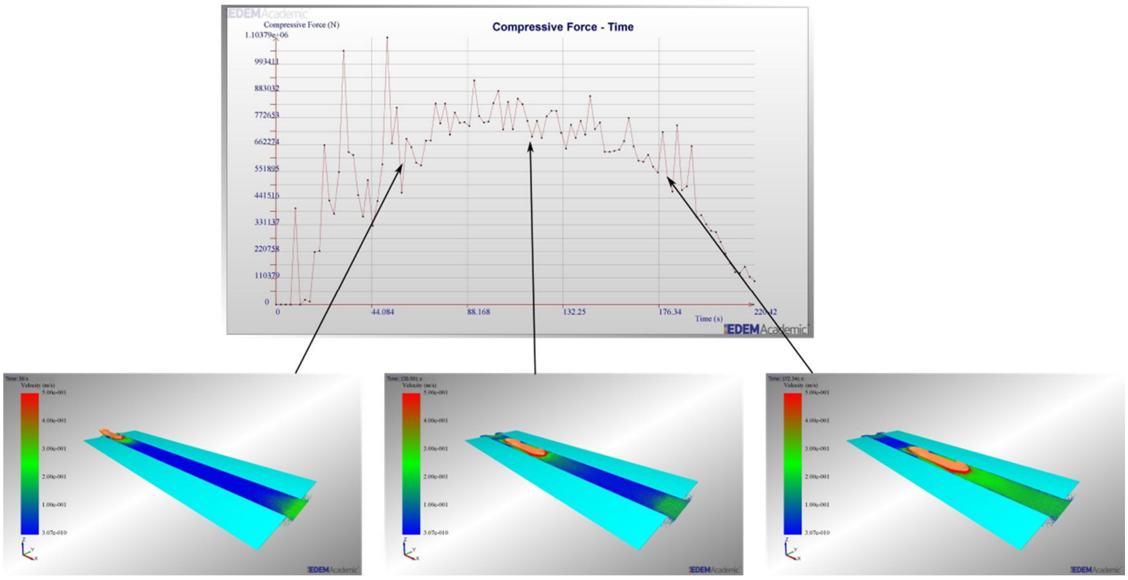


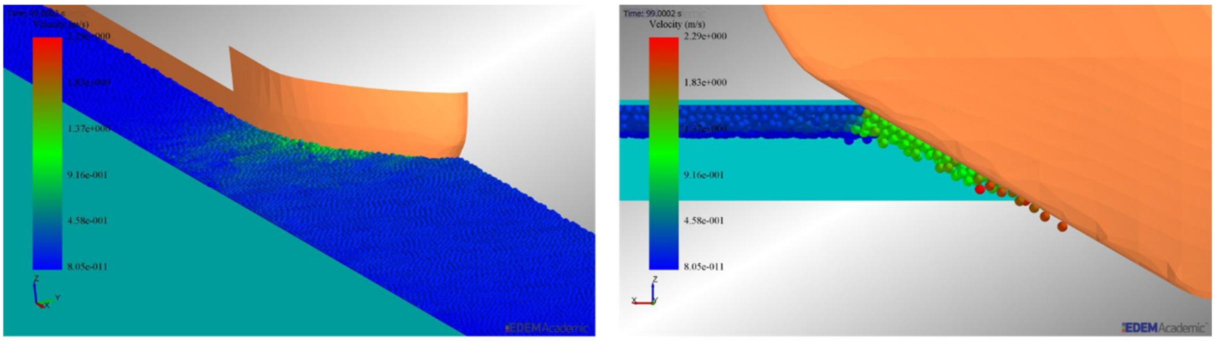
Figure 4. Typical compressive force graph as a function of time.The color distribution indicates the velocity of particles: blue (~0 m/s), green (0.25 m/s), red (>0.5 m/s).

There are three different phases in the simulation: i) ship enters the channel, ii) ship is in steady state and iii) ship exits the channel. In the first phase the compressive force on the ship increases as the ship enters deeper the channel. The second phase is the steady stead phase, where the compressive force on the ship stays approximately at the same level for a long period of time. At this point the ship is located fully within the channel and thus the left side boundary doesn't have any effect. Also, the outflow from the right side of the channel has no effect on the compressive force on ship as the particles are quasi-static in front of the ship. In the final phase the outflow of particles from the right side reaches the front of the ship. As the particles flow in the positive direction with the ship, the compressive force on ship decreases due to lower inertia.

On the basis of the boundary condition study, we concluded that when the channel length is over three times the length of the ship, a steady state phase is reached and thus the channel length is adequate.

In addition a minor investigation on the effects of static friction value between the ice particles was executed as the value of static friction for ice is highly subjective [13]. Changing the value of the static friction on extreme-scale from 0.05 to 0.95 produced approximately 10% difference in the compressive force, which shows that the trade-off of varying the value of static friction around more reasonable 20% produces negligible effect. Therefore the value of static friction for ice-ice contact was fixed to 0.5. Relative high value of friction was assumed to take account also some resistance due to interlocking of ice particles.

Finally, we investigated the behaviour of ice particles in front of the hull. As the ship hull interacts with particles, it generates domino effect where the force subjected to a particle advanced to other particles further away from the ship. Stabilizing this domino effect is one of the challenges in discrete element method, which is typically taken care of by implementing dissipation to particles. Figure 5(a) shows that the dissipation is sufficient so that the interaction between particles directly in front the ship hull doesn't affect the particles far away from the ship hull.



(a) (b)

Figure 5. Behavior of particles in front of the ship hull; blue (~0 m/s), green (1 m/s), red (>2.3 m/s).

In addition the flow of particles under the ship hull was investigated; see Figure 5(b). The movement of the particles is similar as found in other studies. In some cases the ice blocks form wider structure in front of the hull (fake bow), but in the selected case study the height of the brash ice was approximately 1 m, which is not sufficient to form such a fake structure.

2.3 Results

As the ship moves along the channel, it displaces the particles that are in front of it. These particles subject a compressive force to the ship as they are forced to displace from their initial position. Compressive force is comprised of two forces: inertia and friction, which are exerted as normal force on the surface. To compare the simulation results to analytical results, the component of compressive force was calculated that is parallel to the direction if ship translation. Typical force-time signal is illustrated in Figure 6, where the signal is displayed starting from the moment when the ship enters the channel (force increases) and is in steady state phase (average force approximately at the same level). The analytical calculation method provides a single value for the brash ice resistance, which corresponds to the average value of steady state phase in the force time-history, see 2.2.3.

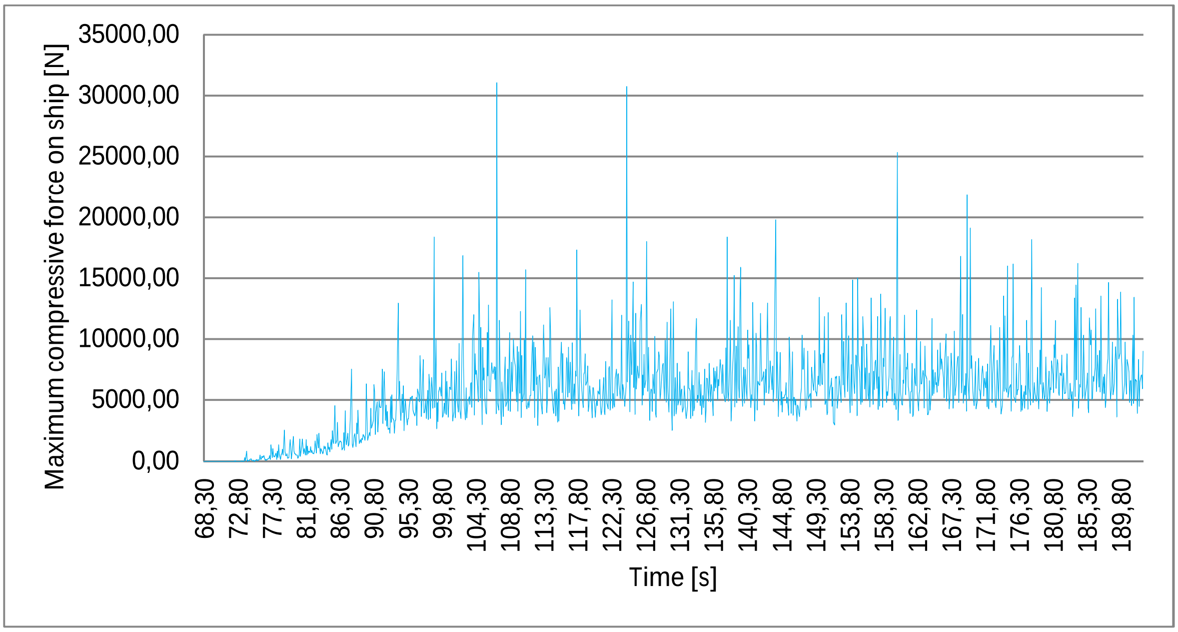


Figure 6. Channel ice resistance (compressive force in x-direction) subjected to the ship as a function of time.

All of the simulations were run with same particle configuration that was generated by placing ice particles randomly within the channel. The porosity of brash ice in the channel configuration was approximately 52%, which was less than measured experimentally [14] indicating relative loose composition of ice particles.

Total of five simulations were ran with different force parameters. In the first run only forces that were active were gravity, buoyancy, inertia and contact forces (incl. friction). In the following simulations fluid drag for particles was implemented in addition to previous forces. The equation used for fluid drag is the following:

$$F_D = \frac{1}{2}\rho v^2 C_D A \tag{1.1}$$

where ρ is the density of water, v is the velocity of the ship, C_D is the drag coefficient and $A = \pi r^2$ is the area of the particle where r is the radius of the particle.

In the last three simulations slight freeze bond between the particles was generated to mimic the cohesive forces found in the reality. The strength of the freeze bond was varied; total of three different values were used based on our experience of ice ridges. For further details on freeze bond and its implementation procedure, see [8].

The results of simulations are given in Table 2. Total of three values were chosen for examination; min-max is the minimum value of all maximum compressive force values during the steady phase, max-max is the maximum value of maximum compressive force encountered during the steady phase and total maximum average value is the average value for maximum compressive force during the steady state phase. The steady state was estimated to be between 120-160 seconds in all the simulations.

Table 2. Summary of simulation results with different parameters with following abbreviation; FD = Fluid Drag, FB = Freeze Bond.

Compressive force	Base simulation	Base + FD	Base + FD + FB (1 MPa)	Base + FD + FB (3 MPa)	Base + FD + FB (100 MPa)
Min-max	2527	3277	1782	3922	3621
Max-max	30729	57359	97683	65533	32441
Total max average	6835	18388	18143	18090	18169

It can be noted that average value for compressive force is for the base simulation 6835 N, and it almost triples to 18143 N when fluid drag was added to the simulation. This is mainly due to increased inertia – as the fluid drag is taken account in the simulations, the particles dislocate more slowly from the front of the hull. This increases the average force subjected to the ship as more force is required to initiate the particles into a movement.

Adding freeze bonds to the simulation didn't produce any considerable effect to the average compressive force as the total average compressive force stays at the same level. It was expected that by adding a cohesive force between the ice particles should increase the compressive force. One reason why this did not occur in the simulations was the particle configuration of brash ice, which was fairly uncompact. As the brash ice composition was created by unisize spherical elements, it turned out to be too porous and therefore weak.

3. Analytical approach

As a comparison for the results obtained using the numerical discrete element method the channel resistance was calculated with two semi-empirical methods both having the basis in the soil mechanics and model test results of ice going vessels. The first is the method developed by Wilhelmson [15] and the second the method described in the Finnish-Swedish ice class rules [4].

3.1 Properties of a typical brash ice channel

The brash ice channel forms typically on the area in which the fairwayl leading to the harbour goes through the fast ice zone. After the traffic has continued regularly for a while, the cross section of the channel will reach gradually a form which is typical for it (Figure 7).

As a consequence of freezing, propeller flow of the vessels and the inner erosion of the ice block mass the ice blocks get rounded and the size of the ice blocks will follow the distribution presented in Figure 8.

The porosity value of brash ice is smaller than that of ridges. Measurements have given values from 0,1 to 0,2 [14].

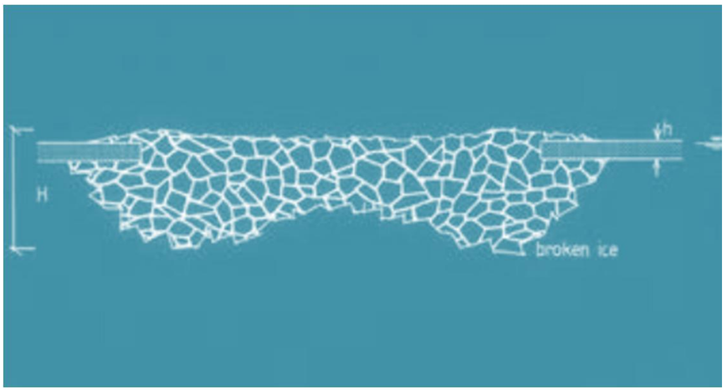


Figure 7. Cross section of a typical brash ice channel [16].

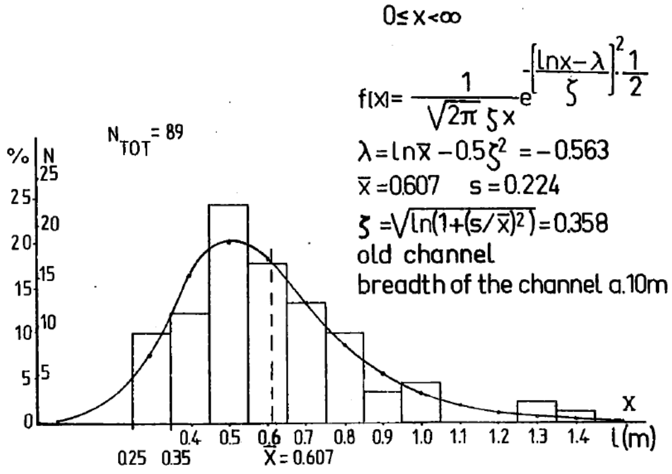


Figure 8. The size distribution of ice blocks in an old channel [17].

3.2 Calculations using semi-empirical methods

The semi-empirical methods used in the calculations were the method developed by Wilhelmson [15] and the method described in the Finnish-Swedish ice class rules [4]. The equations can be seen in the references.

The both methods have their origin in the method developed by Malmberg [18]. In the former method the speed effect to the resistance is included in the equation whereas the latter method is meant for low speed (5 knots) and the equation is simplified for the regulations.

The ship geometry quantities needed for the calculations is the same for the both methods. The quantities are determined in Figure 9 and the numerical values for the Lunni-class vessels are presented in Table 3.

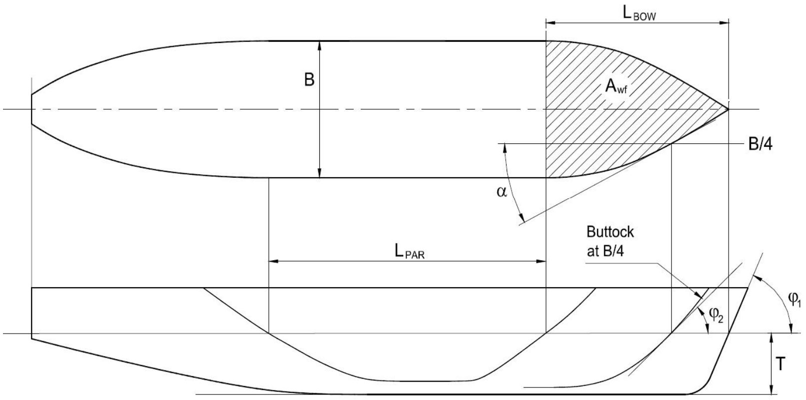


Figure 9. Determination of the geometric quantities of the hull [4].

Table 3. Geometric quantities of the Lunni-class vessels.

Geometric quantity	Symbol	Value
Length of the ship between the perpendiculars	L [m]	150
Length of the bow	L_{BOW} [m]	32,4
Length of the parallel midship body	L_{PAR} [m]	77
Maximum breadth of the ship	B [m]	21,5
Draught	T [m]	9,5
Area of the waterline of the bow	A_{wf} [m^2]	490
The angle of the waterline at B/4	α [°]	24
The rake of the stem at the centerline	$φ_1$ [º]	29
The rake of the bow at B/4	$φ_2$ [º]	29
Flare angle calculated as ψ=arctan(tan φ/sinα)	ψ [º]	54

The thickness of the brash ice in mid channel was taken as 1 m as required in the Finnish-Swedish ice class rules for the ice class IA.

3.3 Results of the analytical calculations

The results of the analytical calculations are presented in Table 4 and in Figure 10.

In the Wilhelmson method the speed effect is included in the resistance in the form of Froude number raised to the power 2. The speed effect has no full scale test background, but in the model tests the effect was noticed to be dependent on the ratio of the main dimensions of the ship. When increasing the speed of the ship from zero to 5 m/s the increase in the resistance is about 30%.

The calculation method of the Finnish-Swedish ice class rule gives about 24% lower resistance value for the speed of 2,5 m/s.

Table 4. Ice resistance in brash ice channel for Lunni-class vesels.

Ship speed [m/s]	Brash ice resistance [kN] Wilhelmson	Brash ice resistance [kN] Finnish-Swedish-ice class rules
0	582	
0,5	584	
1	589	
1,5	598	
2	610	
2,5	626	474
3	645	
3,5	668	
4	695	
4,5	725	
5	758	

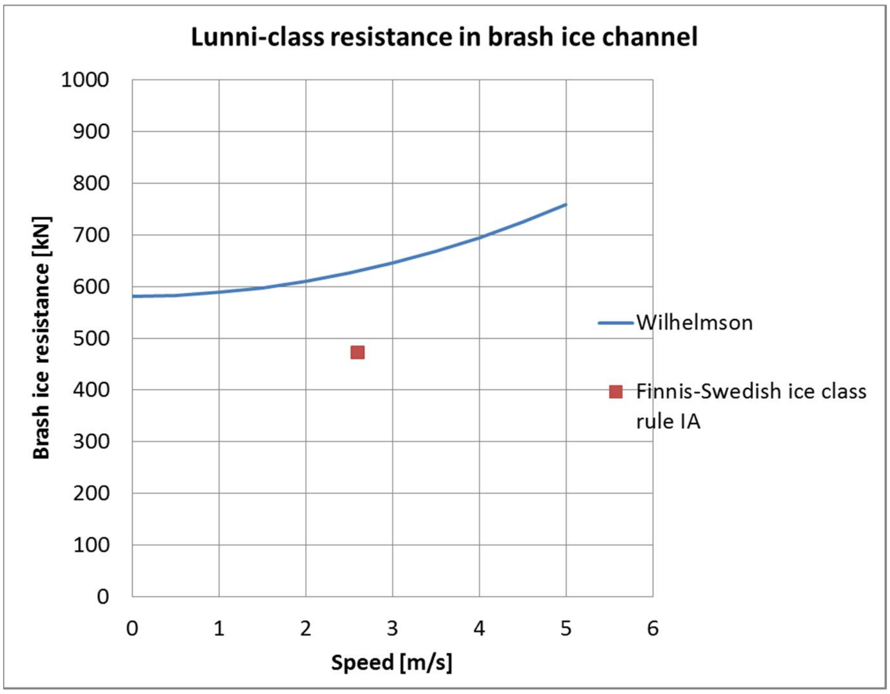


Figure 10. The brash ice channel resistance for the Lunni class. Comparison of results of two methods: the calculation method of the Finnish-Swedish ice class rules [4] and the method developed by Wilhelmson [15].

4. Conclusions and recommendations

The discrete element method was applied to simulate the ice resistance of ship in a channel with brash ice by using EDEM 2.6.0 software. Real ship geometry of Lunni was modelled. In the simulation, the ship was moving unidirectionally in a brash ice channel, which geometry was modelled according to Finnish-Swedish ice class rule. Brash ice in the channel was modelled by a huge number of ice particles with a spherical shape interacting with each other and with the ship.

Reliable boundary conditions, contact parametrization, model size etc. of the model was tested with multiple sanity-checks prior to simulations. Following forces were used in the simulations: gravity, buoyancy, ice to ice contact and friction forces and freeze bonds, ice to ship hull contact and friction forces, inertia and fluid drag of ice blocks.

The simulation of brash ice failure process was visually realistic corresponding to the full-scale observations. However, the channel resistance was substantially lower compared to the analytical study. The major lack in the model relates to the model of brash ice and the evaluation of the material parameters. Current parameterization refers to the ice rubble in ridges. As the brash ice in the channel was created by unisize spherical elements, the porosity was higher than observed by experiments. That made brash ice looser and therefore weaker, which also caused that the implementation of the cohesive freeze bond between the particles didn’t produce any notable effect on the channel resistance.

As the user can model real ship geometry and define the channel geometry and material parameters of ice without considerable restrictions, the DEM-methodology itself is a promising basis for further development. The main needs for future research are focused to measure brash ice properties in full-scale. To understand material behaviour more in details is crucial for reliable channel resistance simulations. For the numerical model validation, corresponding simulations are recommended to be executed with finite element method (FEM). The analysis method used could be similar as in the AZIRULE-project [19], where ABAQUS software with in-house material models and subroutines was applied.

5. References

[1] P. Kujala and T. Sundell, “Performance of Ice-strengthened Ships in the Northern Baltic Sea in Winter 1991,” Helsinki University of Technology, Otaniemi, Espoo, 1992.

[2] P. Lehtinen, “Performance of Ice-Navigating Ships in the Northern Baltic in Winter,” Ship Laboratory, Helsinki University of Technology, Otaniemi, 1993.

[3] P. Lehtinen, “Observations on Ice Navigation Performance of Ships in the Baltic in Winter 1994,” Helsinki University of Technology, Arctic Offshore Research Centre, Otaniemi, Espoo, 1994.

[4] TRAFI, “Maritime Safety Regulation, Ice Class Regulations and the Application Thereof,” TRAFI/31298/03.04.01.00/2010, Helsinki, 2010.

[5] A. Konno, A. Nakane and S. Kanamori, “Validation of Numerical Estimation of Brash Ice Channel Resistance with Model Test,” in *Proceedings of the 22nd International Conference on Port and Ocean Engineering under Arctic Conditions*, Espoo, 2013.

[6] A. Polojärvi, J. Tuhkuri and O. Korkalo, “Comparison and analysis of experimental and virtual laboratory scale punch through tests,” *Cold Regions Science and Technology,* vol. 09, pp. 11-25, 2012.

[7] TRAFI, “Guidelines for the Application of the Finnish-Swedish Ice Class Rules,” TRAFI/21816/03.04.01.01/201, Helsinki, 2011.

[8] A. Sorsimo and J. Heinonen, “Modelling ice ridge punch tests with cohesive 3D discrete element method,” in *22nd IAHR International Symposium on Ice*, Singapore, 2014.

[9] M. Leppäranta, The drift of sea ice, Berlin: Springerverlag , 2011.

[10] G. W. Timco and W. F. Weeks, “A review of the engineering properties of sea ice,” *Cold Regions Science and Technology,* vol. 60, pp. 107-129, 2010.

[11] M. Higa, M. Arakawa and N. Maeno, “Size Dependence of Restitution Coefficients of Ice in Relation to Collision Strength,” *Icarus,* vol. 133, no. 2, pp. 310-320, 1998.

[12] S. Mulmule and J. Dempsey, “Scale effects on sea ice fracture,” *Mechanics of Cohesive-Frictional Materials,* vol. 4, pp. 505-524, 1999.

[13] L. Makkonen and M. Tikanmäki, “Modeling the friction of ice,” *Cold Regions Science and Technology,* vol. 102, pp. 84-93, 2014.

[14] K. Riska, M. Wilhelmson, K. Englund and T. Leiviskä, “Performance of merchant vessels in ice in the Baltic,” Winter navigation research board, Helsinki, 1998.

[15] M. Wilhelmson, ”The resistance of a ship in a brash ice channel,” Helsinki University of Technology, Department of Mechanical Engineering, Master Thesis (in Finnsih), Espoo, 1996.

[16] SSPA, “Brash ice growth - full scale ice tests in the Gulf of Bothnia for the Port of Sabetta,” *SSPA Highlights 58/2013,* 2013.

[17] P. Tuovinen, “The Size Distribution of Ice Blocks in a Broken Channel,” Ship hydrodynamics laboratory, Helsinki University of Technology, Otaniemi, Espoo, 1979.

[18] S. Malmberg, “On the mechanism of getting stuck in ice. (in Swedish),” Helsinki University of Technology. Master Thesis, Espoo, 1983.

[19] A. Kinnunen, M. Tikanmäki, J. Heinonen, J. Kurkela, P. Koskinen and M. Jussila, “Azimuthing thruster ice load calculation,” VTT-R-08842-12, Espoo, 2013.